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Opposite-current flows in gas–liquid layers – III. Non-linear mass transfer

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Abstract

A theoretical analysis of non-linear mass transfer kinetics based on similarity variables method for a gas–liquid opposite-current flow in the conditions of large concentration gradients has been done. The obtained numerical results for the energy dissipation in laminar boundary layers with flat phase boundary and mass transfer rate are compared with analogous results for co-current flows. The ratio between the mass transfer rate and energy dissipation is determined. The induced secondary flow in the gas phase influences mass transfer kinetics significantly when the interphases mass transfer is limited by the mass transfer in gas phase. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Opposite current flow; Non-linear mass transfer; Gas–liquid systems

1. Introduction

In the first two papers [1,2] were found the distribution of the velocities and the concentration at opposite-current gas–liquid flow in the approximation of linear mass-transfer theory when the hydrodynamic and diffusion equations are solved consecutively and independently. In several papers [3,4] was shown that at the conditions of large concentration gradients the secondary flows which velocity depends on the concentration distribution are induced. In this way, the convective diffusion equation becomes non-linear and should be solved in common with Navier–Strokes equations.

2. Mathematical model

The mathematical description of gas–liquid opposite-current flow is shown in [1,2]. At condition of large concentration gradients it is necessary to introduce new boundary conditions [3,4], that express the dependence between the velocity of the induced flows and the concentration gradient:

$$y = 0, \quad v_i = \frac{D_i}{\rho_{0i}^*}\frac{\partial c_i}{\partial y}, \quad i = 1,2. \tag{1}$$

The simultaneous solution of hydrodynamic and diffusion equations in approximation of the boundary layer theory is done after the introduction of similarity variables:

$$\begin{gathered}\eta_i = (-1)^{i+1} y\sqrt{\frac{u_i^\infty}{\upsilon_i l X_i}}, \quad X_1 = \frac{x}{l}, \quad X_2 = \frac{l-x}{l}, \\ X_1 + X_2 = 1, \\ u_i = (-1)^{i+l} u_i^\infty f_i' v_i = (-1)^{i+1}\frac{1}{2}\sqrt{\frac{\upsilon_i u_i^\infty}{l X_i}}(\eta_i f_i' - f_i), \\ f_i = f_i(\eta_i), \\ c_i = c_i^\infty - \chi^{1-i}(c_1^\infty - \chi c_2^\infty)\varphi_i, \quad \varphi_i = \varphi_i(\eta_i), \quad i = 1,2.\end{gathered} \tag{2}$$

In this way, the mathematical model of non-linear mass transfer in systems with intensive interphase mass transfer (large concentration gradients) takes the following form:

$$\begin{gathered}2f_i''' + f_i f_i'' = 0, \quad 2\varphi_i'' + Sc_i f_i \varphi_i' = 0, \\ f_1'(0) = -\theta_1 f_2'(0), \quad \overline{\theta}_2 f_1''(0) = f_2''(0), \\ \varphi_1(0) + \varphi_2(0) = 1, \quad \overline{\theta}_3\varphi_1'(0) + \varphi_2'(0), \quad \varphi_i(\infty) = 0, \\ f_i'(0) = -\theta^{(i)}\varphi_i'(0), \quad i = 1,2,\end{gathered} \tag{3}$$

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Nomenclature

c	concentration (kg mol/m^3)
D	diffusivity (m^2/s)
k	mass transfer coefficient (m/s)
u	velocity in x-direction (m/s)
v	velocity y-direction (m/s)
x	coordinate (m)
y	coordinate (m)

Greek symbols

υ	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
χ	Henry constant

Subscripts

1	for gas
2	for liquid

Superscript

$*$	for co-current flow

where

$$Sc_i = \frac{\upsilon_i}{D_i}, \quad \theta_i = \frac{u_2^\infty}{u_1^\infty}, \quad \theta_2 = \left(\frac{\rho_1\mu_1}{\rho_2\mu_2}\right)^{1/2}\left(\frac{u_1^\infty}{u_2^\infty}\right)^{3/2},$$
$$\theta^{(i)} = \frac{2(c_1^\infty - \chi c_2^\infty)\chi^{1-i}}{\rho_{0i}^* Sc_i}, \quad i = 1,2, \tag{4}$$
$$\overline{\theta}_2 = \theta_2\sqrt{\frac{X_2}{X_1}}, \quad \theta_3 = \chi\frac{D_1}{D_2}\sqrt{\frac{u_1^\infty \upsilon_2}{u_2^\infty \upsilon_1}}, \quad \overline{\theta}_3 = \theta_3\sqrt{\frac{X_2}{X_1}}.$$

In gas–liquid systems was shown [3,5] that non-linear effects in liquid phase may be neglected in comparison to those in gas phase ($\theta^{(2)} = 0$), i.e. they manifest when the mass transfer is limited by the mass transfer in gas phase ($\theta_3 = 0$). At these conditions (3) directly follows $\varphi_2(\eta_2) \equiv 0$, i.e.

$$2f_1''' + f_1 f_1'' = 0, \quad 2\varphi_1^n + Sc_1 f_1 \varphi_1' = 0,$$
$$2f_2''' + f_2 f_2'' = 0,$$
$$f_i(0) = \theta\varphi_i'(0), \quad f_2(0) = 0, \tag{5}$$
$$f_1'(0) = -\theta_1 f_2'(0), \quad \overline{\theta}_2 f_1''(0) = f_2''(0),$$
$$f_1'(\infty) = 1, \quad \varphi_1(0) = 1, \quad \varphi_1(\infty) = 0, \quad i = 1,2,$$

where $\theta = \theta^{(1)}$ determines the direction of the mass transfer in case of absorption ($\theta > 0$) and desorption ($\theta < 0$).

Table 1
Numerical results of the boundary conditions

θ	x_1	$\theta_1 = 0.1$			$\theta_2 = 0.152$		
		$f_1'(0)$	$f_1''(0)$	$\varphi_1'(0)$	$f_1'(6)$	$f_2'(6)$	$\varphi_1(6)$
$\theta = 0$	0.5	−0.090800	0.327598	−0.30035	0.998970	0.998984	0.000872
$\theta = 0.1$	0.05	−0.051580	0.341510	−0.32470	0.998964	0.998956	0.001167
	0.1	−0.069200	0.340400	−0.31850	0.998853	0.999076	0.000660
	0.2	−0.080330	0.339550	−0.31380	0.998945	0.998994	0.002572
	0.3	−0.085230	0.339150	−0.31260	0.998978	0.998982	0.000668
	0.4	−0.088271	0.338860	−0.31140	0.998977	0.998976	0.000857
	0.5	−0.090480	0.338640	−0.31060	0.998971	0.998955	0.000757
	0.6	−0.092259	0.338460	−0.30980	0.998980	0.998966	0.001164
	0.7	−0.093805	0.338300	−0.30930	0.998974	0.998943	0.000896
	0.8	−0.095270	0.338144	−0.30872	0.998973	0.998962	0.000973
	0.9	−0.096830	0.337975	−0.30815	0.998969	0.998969	0.000900
$\theta = -0.1$	0.05	−0.05540	0.320120	−0.30330	0.998975	0.998977	0.001025
	0.1	−0.07137	0.318910	−0.29770	0.998972	0.998933	0.000844
	0.2	−0.08166	0.317955	−0.29390	0.998960	0.998963	0.001135
	0.3	−0.08621	0.317493	−0.29220	0.998966	0.998944	0.001274
	0.4	−0.08904	0.317190	−0.29120	0.998971	0.998944	0.001138
	0.5	−0.09110	0.316963	−0.29040	0.998972	0.998943	0.001282
	0.6	−0.09276	0.316775	−0.28977	0.998971	0.998963	0.001339
	0.7	−0.09420	0.316605	−0.28930	0.998970	0.998914	0.001110
	0.8	−0.09557	0.316446	−0.28880	0.998974	0.998951	0.001080
	0.9	−0.09703	0.316276	−0.28813	0.998974	0.998981	0.001532

Table 2
Numerical results of the boundary conditions

θ	x_1	$\theta_1 = 0.1$			$\theta_2 = 0.152$		
		$f_1'(0)$	$f_1''(0)$	$\varphi_1'(0)$	$f_1'(6)$	$f_2'(6)$	$\varphi_1(6)$
$\theta = 0.2$	0.05	−0.049490	0.352790	−0.33620	0.998978	0.998991	0.000836
	0.1	−0.068000	0.351830	−0.32940	0.998985	0.998933	0.001038
	0.2	−0.079610	0.351000	−0.32520	0.998964	0.998931	0.000706
	0.3	−0.084700	0.350590	−0.32330	0.998980	0.998916	0.000674
	0.4	−0.087850	0.350300	−0.32200	0.998968	0.998869	0.000983
	0.5	−0.090150	0.350100	−0.32120	0.998980	0.998950	0.000775
	0.6	−0.091990	0.349920	−0.32050	0.998968	0.998934	0.000765
	0.7	−0.093590	0.349760	−0.31977	0.998974	0.998921	0.001117
	0.8	−0.095110	0.349614	−0.31930	0.998969	0.998956	0.000785
	0.9	−0.096718	0.349450	−0.31860	0.998977	0.998902	0.001034
$\theta = -0.2$	0.05	−0.057140	0.309970	−0.29320	0.998981	0.998919	0.001016
	0.1	−0.072400	0.308700	−0.28780	0.998960	0.998987	0.001175
	0.2	−0.082290	0.307723	−0.28428	0.998968	0.998992	0.001115
	0.3	−0.086670	0.307260	−0.28261	0.998980	0.998921	0.001446
	0.4	−0.089400	0.306940	−0.28168	0.998969	0.099891	0.001200
	0.5	−0.091389	0.306710	−0.28094	0.998971	0.998906	0.001253
	0.6	−0.092990	0.306530	−0.28030	0.998985	0.998902	0.001466
	0.7	−0.094390	0.306350	−0.27982	0.998969	0.998936	0.001326
	0.8	−0.095712	0.306190	−0.27935	0.998979	0.998950	0.001268
	0.9	−0.097120	0.306020	−0.27880	0.998987	0.998943	0.001391

Table 3
Numerical results of the boundary conditions

θ	x_1	$\theta_1 = 0.1$			$\theta_2 = 0.152$		
		$f_1'(0)$	$f_1''(0)$	$\varphi_1'(0)$	$f_1'(6)$	$f_2'(6)$	$\varphi_1(6)$
$\theta = 0.3$	0.05	−0.047250	0.36446	−0.3481	0.998982	0.998968	0.000731
	0.1	−0.066750	0.36362	−0.3410	0.998971	0.998809	0.000574
	0.2	−0.078870	0.36285	−0.3364	0.998978	0.998952	0.000722
	0.3	−0.084160	0.36245	−0.3343	0.998988	0.998955	0.000975
	0.4	−0.087433	0.36218	−0.3331	0.998968	0.998959	0.000827
	0.5	−0.089810	0.36198	−0.3322	0.998968	0.998974	0.000794
	0.6	−0.091720	0.36180	−0.3314	0.998958	0.998990	0.000966
	0.7	−0.093378	0.36167	−0.3309	0.998968	0.998972	0.000588
	0.8	−0.094945	0.36152	−0.3302	0.998977	0.998963	0.000849
	0.9	−0.096616	0.36135	−0.3295	0.998968	0.998973	0.000989
$\theta = -0.3$	0.05	−0.058800	0.30018	−0.28330	0.998975	0.998982	0.001707
	0.1	−0.073340	0.29886	−0.27839	0.998973	0.998669	0.001134
	0.2	−0.082290	0.29786	−0.27496	0.998971	0.998981	0.001315
	0.3	−0.087123	0.29738	−0.27340	0.998970	0.998996	0.001487
	0.4	−0.089755	0.29707	−0.27243	0.998971	0.998960	0.001576
	0.5	−0.091678	0.29683	−0.27175	0.998966	0.998976	0.001509
	0.6	−0.093223	0.29664	−0.27120	0.998974	0.998954	0.001471
	0.7	−0.094570	0.29647	−0.27071	0.998974	0.998929	0.001472
	0.8	−0.095851	0.29631	−0.27024	0.998983	0.998971	0.001491
	0.9	−0.097213	0.29613	−0.26972	0.998973	0.998969	0.001576

3. Numerical results

Problem (5) was solved for the following values of the parameters:

$$Sc_1 = 1, \quad \theta_1 = 0.1, \quad \theta_2 = 0.152, \\ \theta_3 = \pm 0.1, \pm 0.2, \pm 0.3. \tag{6}$$

For this purpose, the boundary contains were introduced:

$$f_1(0) = \alpha, \quad \varphi_1'(0) = \frac{\alpha}{\theta}, \\ f_1'(0) = \beta, \quad f_2'(0) = -\frac{\beta}{\theta_1}, \\ f_1''(0) = \gamma, \quad f_2''(0) = \overline{\theta}_2\gamma,. \tag{7}$$

where α, β, γ are varied so that after the solution of (5) the following boundary conditions are to be obtained:

$$f_1'(\infty) = 1, \quad f_2'(\infty) = 1, \quad \varphi_1(\infty) = 0. \tag{8}$$

The numerical realization of this method was done in [1,2] and the obtained results are shown in Tables 1–3, where the new boundary conditions are presented:

$$\alpha = \theta\varphi_1'(0), \quad \beta = f_1'(0), \quad \gamma = f_1''(0), \quad f_1'(\infty) = f_1'(6), \\ f_2'(\infty) = f_2'(6), \quad \varphi_1(\infty) = \varphi_1(6). \tag{9}$$

The obtained results (9) are in compliance with the boundary layer theory [6], where the velocity reaches approximately its asymptotic value at $\eta_i > 5$ $(i = 1, 2)$, and the thicknesses of the hydrodynamic and diffusion boundary layer are from the same order.

The solution of the problem in case of co-current flow was obtained directly from (5) for $\theta_1 = -0.1$ and the obtained results are shown in Table 4.

4. Energy dissipation and mass transfer kinetics

In [1] was shown, that the energy dissipation may be determined for a counter-current flow (E) and for a co-current one (E^*):

$$E_i = \int_0^1 \frac{1}{\sqrt{X_i}} \left[\int_0^\infty (f_i'')^2 \, d\eta_i \right] dX_i, \\ E_i^* = 2 \int_0^\infty (f_i''^*)^2 \, d\eta_i, \quad i = 1, 2. \tag{10}$$

In case of non-linear mass transfer in gas phase the results are shown in Table 5. The rate of mass transfer [3,4] is determined from the Sherwood number:

$$Sh_1 = \frac{\rho^*}{\rho_0^*} \sqrt{Re_i} J_i, \quad J_1 = -\int_0^1 \frac{\varphi_i'(0)}{\sqrt{X_1}} \, dX_1, \quad Re_i = \frac{u_i^* l}{v_i}. \tag{11}$$

In case of co-current flow:

$$J_1^* = -2\varphi_1'(0) \tag{12}$$

The obtained results for J_1 and J_1^* are shown in Table 5, where the ratio $A = J/E$ presents the mass transfer energy efficiency (mass transfer rate in result of energy dissipation).

$$A_1 = \frac{J_1}{E_1}, \qquad A_1^* = \frac{J_1^*}{E_1^*}. \tag{13}$$

Table 4
Numerical results for co-current flow

θ	$\theta_1 = -0.1$			$\theta_2 = 0.152$		
	$f_1'^*(0)$	$f_1''^*(0)$	$\varphi_1'^*(0)$	$f_1'^*(6)$	$f_2'^*(6)$	$\varphi_1^*(6)$
0	0.090800	0.32765	−0.3604	0.998982	0.998990	0.001041
0.1	0.090513	0.33753	−0.3713	0.998972	0.998965	0.000649
−0.1	0.091070	0.31812	−0.3502	0.998984	0.998972	0.000528
0.2	0.090220	0.34799	−0.3825	0.998970	0.998984	0.000546
−0.2	0.091330	0.30892	−0.3403	0.998977	0.998945	0.000154
0.3	0.089910	0.35843	−0.3941	0.998972	0.998951	0.000470
−0.3	0.091580	0.30006	−0.3306	0.998983	0.998918	0.000242

Table 5
Energy dissipation, mass transfer rate and mass transfer energy efficiency

θ	E_1	E_1^*	J_1	J_1^*	A_1	A_1^*
0.3	0.544	0.477	0.616	0.788	1.13	1.65
0.2	0.537	0.471	0.595	0.765	1.11	1.62
0.1	0.529	0.464	0.575	0.743	1.09	1.60
0.0	0.525	0.458	0.554	0.720	1.05	1.57
−0.1	0.516	0.452	0.538	0.700	1.04	1.55
−0.2	0.509	0.446	0.520	0.681	1.02	1.53
−0.3	0.503	0.441	0.503	0.661	1.00	1.50

5. Conclusion

The results from numerical experiments (Table 5) show that energy dissipation E_1 at absorption ($\theta > 0$) is higher than the one obtained [2] in linear approximation ($\theta = 0$). At condition of desorption ($\theta < 0$) the relation is opposite. The dependence of the rate of a diffusion transport (average diffusion flux J_1) and the mass transfer energy efficiency (A_1) is analogous, and at the absorption (desorption) they are larger in comparison to the linear approximation $\theta = 0$ in [2]. These effects increase at the increase of concentration gradient (absolute value of θ).

The obtained results show that the co-current flow regime is more efficient energetically than the counter-current one.

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